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#258: The world by numbers: How mathematics explains objects and events

VOICEOVER
Welcome to Up Close, the research talk show from the University of Melbourne, Australia.

SHANE HUNTINGTON
I'm Dr Shane Huntington. Thanks for joining us. Mathematical descriptions of physical and biological processes are often described as elegant. But they are rarely simple. Often equations that appear quite simple only hold true for very specific conditions. When we start to introduce the complexities of the real world, more complex mathematical approaches need to be applied ? approaches that are far from the equations the likes of Sir Isaac Newton would have been familiar with. Many of the problems tackled by mathematicians today involve multiple properties, multiple objects and multiple potential interactions between those objects. In some cases, subtle changes in approximations made by a mathematician can result in significant differences in predictions. To describe the approach taken by mathematicians in solving real world problems, and to explore some of the equations used, we are joined today on Up Close by Professor Chris Budd, Professor Applied Mathematics at the University of Bath and Professor of Mathematics at the Royal Institution of Great Britain. Professor Chris Budd is in Melbourne to speak at the Maths of Planet Earth Australia Conference, organised by the Australian Mathematical Sciences Institute along with the Academy of Science and the Australian Research Council. Welcome to Up Close, Chris.

CHRIS BUDD
Hello.

SHANE HUNTINGTON
There are many types of equations used by scientists, engineers, economists and other specialists. Today we're talking about differential equations in particular. Can you define what these are for us?

CHRIS BUDD

Yes, a differential equation is a description about how something changes. So, if I have an object such as the velocity of the wind, for example, then that changes in time. It also changes from one place to another. What a differential equation does is it tells you how big the change is.

SHANE HUNTINGTON
Chris, there are many different types of equations that people will encounter. How do differential equations contrast themselves to those other equations we'd find in various areas of economics, the weather and so forth?

CHRIS BUDD
Well, the key thing about a differential equation is it's explaining how something actually evolves and changes. So, if something is changing, you need a way of understanding that change and that's what a differential equation does. So it's different from, say, a quadratic equation which gives an exact description of how something is at a particular time. It's the change which is important. As change is all about us and everything we do involves change, this is why differential equations are so useful in understanding the world.

SHANE HUNTINGTON
Now, in physics one of the first problems that we would teach students is how to calculate an object's position and velocity?

CHRIS BUDD
Yes.

SHANE HUNTINGTON
?at a particular time. Could you talk us through an example of this? Say, for example, an object falling in the Earth's atmosphere, how would we go about solving that problem.

CHRIS BUDD
Yes. Well, a good example would be, for example, a rugby ball which you've kicked and it's moving through space. So its velocity is changing because it's being accelerated towards the Earth by the force of gravity. That gives you a differential equation for its vertical velocity. At the same time, the ball is probably moving horizontally as well. You have a separate differential equation describing that. The equations take into account a number of things. They take into account gravity but also the resistance on the rugby ball and maybe other factors such as the spin of the ball.

SHANE HUNTINGTON
Now, there are a number of things happening there that presumably are changing in time themselves, not only the ball's position but the way in which those other interactions are occurring on the ball. Do the differential equations take those changes into account as well?

CHRIS BUDD
Well, they do. I mean, when you kick a rugby ball, the first thing that happens is you apply force to it and give it some initial velocity. As it moves, so its position and velocity themselves change. Also maybe there's wind blowing across the rugby field and that will also change and that will interact with the ball itself. So all of these factors need to be taken into account.

SHANE HUNTINGTON
How do we go about solving these particular equations? What sort of answers do they give us? Do they give us accurate answers or do we have a scenario where we essentially are getting probabilities?

CHRIS BUDD
That's a really good question. It really depends on the system. Perhaps the first system to be properly understood and described in terms of differential equations was the motion of the planets around the Sun. In that case, you can write down very precise equations. In certain cases such as the Earth moving around the sun, you can solve them exactly in terms of mathematical formulae. Those formulae predict what you find, which is that the Earth goes around the Sun in an ellipse. Now, that's kind of unusual. More normally equations for something such as the weather are much, much more complicated. It's very hard to find an exact solution of them. As you said, there might even be a sort of probabilistic solution because of all the things that you don?t know and you have to estimate. What you then do is you put those equations into a computer and the computer helps you solve them.

SHANE HUNTINGTON
Now, I want to dive into this area of finding a solution a bit deeper. You referred to this a couple of times. Once we have the equation written down, the differential equation for the object in motion?

CHRIS BUDD
Yes.

SHANE HUNTINGTON
?is that not the solution? What do we have to do that from point to make this useful?

CHRIS BUDD
Well, again, that's a good question. To a physicist, you might say, ?well, I have a solution,? because the maths is completely describing what's going on. But, at the same time, you still need to work out what it's going to do. For example, if I had an equation like $X + 2 = 5$, I still need to find X. I subtract two from both sides and I find $X = 3$. With a differential equation, it's kind of like that. You might have an equation for some quantity such as X, which might be the position of the rugby ball, but you still have to kind of untangle that to find the actual solution so you know actually what it's doing. Really that's kind of hard. The first equations that were written down, which were the ones that Newton wrote down, he was incredibly lucky actually that he was looking at a system where the maths of that time could actually find a solution. Most

of the equations that we look at, for even kind of very everyday things, such as the flow of water, are really too tough to find easy solutions for. This is why a computer is needed to actually make exact predictions.

SHANE HUNTINGTON
When we go into these areas of complexity, presumably there is some value in knowing what all the dependencies are to write down that first equation. Can we get much from that or do we have to solve the equation before it's of use?

CHRIS BUDD
It's very important if you have lots of parameters in the system, for example, the temperature or the wind blowing on the ball, to know how that affects the actual behaviour. A lot of mathematics gives you answers to how things depend on these quantities even if you can't solve it exactly. One example would be the drag on a rugby ball is known to be proportional to the square of its speed. That gives you a lot of insight into how it then moves.

SHANE HUNTINGTON
Now, you mentioned some of these complex systems. How complex do these equations themselves get? How easy are they to solve? We start teaching students to solve these equations in late high school, first years of university. Are these the sorts of equations that students at that point would be able to solve or are they well beyond that?

CHRIS BUDD
Well, let's give two examples. Going back to the example of the Earth going around the sun, you have two equations which completely describe it, an equation for its position and an equation for its velocity. Those two equations can be solved exactly and they can be solved by a high school student. They are solved by high school students. If you take something like the weather, in contrast, you have, we reckon, about a billion equations for the weather. Those billion equations couldn?t be solved by anyone. That's why we need something like a computer that can cope with the complexity of that system.

SHANE HUNTINGTON
Chris, some of our listeners may have heard the term partial differential equations.

CHRIS BUDD
Right.

SHANE HUNTINGTON
Are these differential equations that aren?t playing ball? What does that mean?

CHRIS BUDD
If you have a quantity like the wind, then the velocity of the wind depends, not just where you are in space, but also what the time is. It depends on two things, space and time. A partial differential equation is an equation about something which

depends on space and on time.

SHANE HUNTINGTON
You're listening to Up Close. In this episode, we're talking about differential equations with mathematician, Chris Budd. I'm Shane Huntington. Chris, we've mentioned a number of the very simple scenarios. What about if we move towards something like a car accident or significant impacts where there is deformation and so forth? What sort of equations and how do we go about solving those equations for those particular areas of interest?

CHRIS BUDD
Well, let's start with a bit of context, first of all. One of the most useful applications of what I'm talking about is the process of understanding what happens in a car accident as it allows the police and the forensic people to see what happens and what the causes and effects were. So this is really important mathematics. It's also used to make cars safer and to make the way that say, for example, seatbelts and collision stuff within the car is designed. So this really does make things safer for us. Now, the equations for cars impacting on each other are not too complex. Again, you need an equation for the speed and velocity of the car. When they come into contact, you have equations for the way the metal deforms under impact and other aspects of that such as the way that the human being might impact with the car itself and how unfortunately they deform on impact as well. So these systems are reasonably straightforward to study. The easiest example to have in mind would be, say, dropping a ball on the ground and letting it bounce back. That is a system where we can write down more or less what's going on. Again, in a kind of really complex scenario, you have to do quite a lot of work to see exactly what was happening. If you want to estimate something like the speed of the car just before impact, which is actually rather important in a criminal trial, then you can do that more or less by solving exact equations.

SHANE HUNTINGTON
Now, let's talk a bit about the impact because one of the things that many of our listeners would have noticed throughout their lives is there has been quite a change in the design of cars?
CHRIS BUDD
Yes.

SHANE HUNTINGTON
?from cars that are very solid, often made of very hard materials, to cars that crumple.

CHRIS BUDD
Yes.

SHANE HUNTINGTON
Mathematically, why is it important that a car has these crumple zones and is able to deform?

CHRIS BUDD
Well, basically the effect of a collision on the body is proportional to the energy that you have in the system but also is inversely proportional to the time over which that energy is dissipated. So, if you dissipate the energy over a very short time, that has a very severe effect on you. If you spread that out over a much longer time, then it has much less effect. That's the advantage for having a crumple zone. But one thing that's worth noting is that the amount of energy in a collision is proportional to the square of your speed. So, if you double your speed, the actual energy goes up by a factor of four. If you triple it, it goes by a factor of nine. This is why high-speed collisions are much, much more dangerous than low speed ones.

SHANE HUNTINGTON
Mathematical predictions of the type you're speaking of, on paper seem relatively easy to put down. How useful are predictions when we move them into the real world? Do they start to very rapidly lose their voracity when we move into real situations where there are far more parameters to be concerned about? Or is the validity still there?

CHRIS BUDD
The validity is still there. I'll give you an example. Modern aircraft are invariably designed not using wind tunnels but by doing calculations. So an aircraft will be designed by solving the equations for air flow over the fuselage in the wings. Those equations give you a huge amount of information as to how that aeroplane will behave. All the main calculations are now done by solving these equations. They may only do a wind tunnel test right at the very, very end of the process. So these are incredibly powerful and accurate calculations. The complexity doesn't mean you can't do it. It just means that you have to work a lot harder to do it.

SHANE HUNTINGTON
Chris, give us an idea into your sort of day when you take on a new problem. At what point does the scribbling on the page of equations get taken over by the computer modelling? When does that occur?

CHRIS BUDD
It occurs right at the beginning. What I normally do is develop equations at the same time as I put them onto the computer. So the computer is used in the early stages to give me insight to make sure I'm heading in the right direction and at a later stage to really get to grips with the big calculations. So I will be scribbling on paper and on a computer almost at the same time simultaneously. Modern technology is so good that you can get equations onto computer very quickly and play with them almost immediately.
SHANE HUNTINGTON
When we look at particular problems where the outputs of the equations and the computer work are dictated very strongly by probabilities, by ideas of errors?

CHRIS BUDD
Yes.

SHANE HUNTINGTON
?what is the distinguishing factor that makes these problems so difficult to nail down with the level of accuracy compared to other problems, as you mentioned before like the position of the planets where we can quite easily and quite readily predict?

CHRIS BUDD
Well, with the planets, you've got a lot of precise information. You don?t have too many things going on. A problem like the weather is much, much harder. It's harder for two reasons. First, there's a lot more going on. Also, there's a lot less that we know. In particular, we don?t really know the initial state of the atmosphere very well. So what is often done in a subject like weather forecasting is you might, instead of starting the weather off from one configuration predicting what's going to happen in, say, three days' time, you might make multiple calculations where you take a whole load of different initial configurations, which will differ from each other by a small amount consistent with errors in measurements. Then you forecast what's called an ensemble of forecasts three days into the future. Now, if that ensemble of forecasts all basically agree with each other, then you say, yes, with high confidence that's what's going to go on. If one-third did one thing, two-thirds said another, you might then say, well, there's a two-thirds chance it's going to rain and a one-third chance that it doesn't. That's how you get probabilities. You often find in modern weather forecasts that you do get weather with certain probabilities attached. That's basically how it's done.

SHANE HUNTINGTON
Chris, let's now move into one of the more intriguing areas of mathematics which was first described by Edward Lorenz, namely chaos theory. Can you explain for us what the particular group of problems is that chaos theory describes?

CHRIS BUDD
Chaos theory really is a description of systems which are what's called non-linear. So a non-linear system is where you have different things which interact very closely. What you get out isn?t the sum of those interactions. It's a much more complicated combination of them. A good example of a non-linear system would be the weather itself. Lorenz, who you referred to, was a meteorologist.

SHANE HUNTINGTON
When we take a chaos problem, how does it compare to the sorts of differential equation problems we were looking at earlier? What's different about the parameters that we're putting in and the outcomes that we will get?

CHRIS BUDD
Well, if you take a system which is not chaotic, which is the Earth going around the Sun, then, if you make a small change to the Earth's position or a small change to its mass or something like that, then that actually doesn't affect the solution very much. A small change in the configuration leads to a small change in the solution. That's an example of a non-chaotic system. Now, we all know that the weather is rather unpredictable. One of the reasons it's unpredictable is that quite small changes to the

initial states or to the parameters in the equations can, over quite a short time, lead to very large changes in the solution. That's the hallmark of a chaotic system. So chaos means really unpredictable behaviour largely arising because you get big changes due to small effects.

SHANE HUNTINGTON
When we consider something like the weather where there are so many measurements being made to go into these particular theories, and each of those measurements has errors associated with it, it is quite stunning that we can predict the weather to such accuracy, it would seem, in a chaotic system.

CHRIS BUDD
Well, the reason for that is, over short times, the affects in errors in the measurements don?t have too much effect. The weather tomorrow is with a chance of about 70 per cent what the weather is today. The effects of errors accumulate. That accumulation means that, after about 10 days, you basically can't predict the weather at all. It is certainly true though that the weather is very sensitive to these things. It is still remarkable that we can get anywhere.

SHANE HUNTINGTON
I'm Shane Huntington. My guest today is mathematician, Chris Budd. We're talking about mathematical predictions here on Up Close. Chris, coming back to our initial discussion on types of equations?

CHRIS BUDD
Yes.

SHANE HUNTINGTON
?you mentioned quadratic equations, differential equations. When we're dealing with something like chaos theory, what sort of equations are involved with those particular predictions?

CHRIS BUDD
Well, there are two types of equations which are very popular in chaos theory. The first is the ones I've been describing which are differential equations. The other type of equation which comes up in a lot of cases are what are called maps. Now, I'll give you an example of a map. Let's suppose we know what the population of Melbourne is in this year, 2013. What's the population going to be in 2014? What's the population going to be in 2015? What's it going to be in 2030? Now, there might be a sort of relationship between the population this year and the population next year. That relationship is called a map. Maps are used quite a lot by biologists to understand the way populations of animals grow and evolve from one generation to the next. A very famous map is called the logistic map which was basically derived to look at the way populations change or how disease propagates. This map is known to be chaotic and to give somewhat unpredictable types of behaviour.

SHANE HUNTINGTON

One of the areas that chaos theory would seem to bring into strong focus is the desire and need for extraordinary computing power. How has that changed the way we do chaos modelling over the last sort of decade as we've exploded our capacity in computing power?

CHRIS BUDD
Well, I think it's fair to say that the discovery of chaos more or less coincided with the first use of computers to solve equations. So Lorenz wrote down his equations in about 1965. This was when computers were starting to be used to solve differential equations. It was because they applied computer-used equations and found things which they completely didn?t understand or predict that the whole subject erupted. Now, because computers are so powerful and so universally used, we kind of have chaos automatically built into a lot of the sort of problems that we work with.

SHANE HUNTINGTON
When we look at something as substantial in a social sense as climate change and we consider that this is based on chaos theory and based on many initial predictions and many initial measurements, how much can we rely on the outcomes given when you mentioned weather, you said, beyond 10 days things are difficult? Obviously these are different, larger systems. But where's the reliability there?

CHRIS BUDD
Well, it's difficult to predict the weather 10 days ahead because of the effects of chaos. But climate's rather different. Climate is much more about the kind of slow evolution of average quantities rather than predicting what's going on from day to day. The equations for this are somewhat different and much, in a sense, more predictable. So we can understand how the Earth's temperature is going to arise year on year due to climate change even if I can't tell you whether it's going to rain or not on Christmas Day.

SHANE HUNTINGTON
With that, is the application of chaos theory still happening or is it back to the sort of more standard equations where those initial conditions aren?t as pertinent?

CHRIS BUDD
Oh no, chaos theory is hugely important in many, many areas of science across the board, from physics through to biology. The reason for that is chaos is there. Chaos is a natural occurring physical phenomenon. The simplest system I know which is chaotic is what's called a double pendulum, which is one pendulum tied onto the bottom of another pendulum so they can oscillate together. That system behaves beautifully chaotically. It has completely unpredictable behaviour. It's there. It's there in the world about us. We cannot avoid it. What we do is we take it on board. It's just another aspect of science which we try to understand.

SHANE HUNTINGTON
When you refer to that as unpredictable, does chaos theory itself not give us an insight into what it will do? Or is it inherently non-predictable?

CHRIS BUDD
What chaos theory does is it says there may be systems out there which look as though they are unpredictable. But what chaos theory does is say, let's look a bit deeper. Maybe underneath those seemingly unpredictable systems are equations which we can write down and, by solving them, get an insight into the system. That's really been the great breakthrough. So the things which we thought in the past were completely random we now know are not random. They are described by equations. It's just that the equations themselves have somewhat crazy behaviour.

SHANE HUNTINGTON
You mentioned the airline industry. What other industries are mathematicians at the moment getting heavily involved with, yourself, for example? Where are the outcomes from those particular projects?

CHRIS BUDD
Well, I think it's fair to say that maths gets into just about every industry going. The biggest industry I am familiar with which uses maths is the information industry. So Google and the internet heavily rely on maths. Chaos itself is important in areas such as the electronics industry. The electronics industry uses an enormous amount of maths. But it comes into almost anything including, for example, the food industry is a big, big user of mathematics.

SHANE HUNTINGTON
Chris, is maths a science in itself or is it a service to other sciences?

CHRIS BUDD
I like to think of maths as just something on its own. It's not a science. It's not an art. It's just mathematics.

SHANE HUNTINGTON
Chris, you mentioned Google, which is very much a human construct. That brings us to the issue of human behaviour.

CHRIS BUDD
Yes.

SHANE HUNTINGTON
Is this something that chaos theory has a say in or are we very predictable?

CHRIS BUDD
Well, I don?t think any human is particularly predictable. The brain is such an amazing thing and such a complex thing. I certainly couldn?t predict the behaviour of any one human being. I can't even predict the behaviour of my dog. One thing that we can use chaos theory for, and it is powerfully used actually, is to understand the behaviour of humans in large crowds and the way people interact in crowds can be described using differential equations. These equations are incredibly useful when designing things like sports stadia, railway stations or simply understanding how

people might walk down the street.

SHANE HUNTINGTON
So, when you do look at something like a stadium, what sort of mathematics is involved there? What sort of things are you trying to predict, or determine or design?

CHRIS BUDD
If you want to understand how a crowd moves, let's suppose you have a worst case scenario and there's a fire in the stadium. Then an individual will obviously want to leave the stadium through an exit. They will have some desire to move in a certain way towards that exit. They?ll also want to avoid other people so they don?t want to collide into those people. If you're there with family members then you will want to gather those members with you and walk together with them. The equations take into account this desire, where we want to go, our desire to avoid people, our desire to keep families with us and the fact that we can't walk through walls and stuff like that. Put all those together, you get a system of equations which you can kind of use to simulate the motion of the people. Then that is used by people designing sports stadia to work out where to locate the exits, how wide the exits should be and even what signage there should be towards the exits.

SHANE HUNTINGTON
Presumably, Chris, many of the equations and techniques we're talking about are also used in the gaming industry. By that I mean the online and non-online virtual environments that are used for entertainment and where virtual environments are designed that have to appear and interact with the users as though they are real and follow the same rules.

CHRIS BUDD
Well, very much so. One of the things I love telling my students is one of the largest single employers of mathematicians is the gaming industry because of exactly what you say. They want to have reality, virtual reality things that look as close to reality as possible. That requires a lot of maths, both in understanding how things move, which is back to differential equations, or how light comes off them. Well, that's differential equations as well.

SHANE HUNTINGTON
Chris, when we consider some of the equations that we're talking about that cover aspects of the natural world we exist in, do these equations exist independent of us? Are they real things or are they just constructs of our own minds?

CHRIS BUDD
This is one of the great philosophical debates in mathematics. Is mathematics an invention or is it a discovery? It's incredible that mathematics can be used in this way to describe the world. It's got an unreasonable effectiveness. It does very much seem as though differential equations are our best way of understanding the world and that the world's kind of, in a sense, is differential equations. A really good example of that is quantum theory, which is all based on a single equation called

Schrödinger's equation which, at a very, very kind of fundamental level, does seem to describe the world that we're in. Whether there might be something deeper than that going on underneath, well, that's really a question for philosophers rather than mathematicians.

SHANE HUNTINGTON
We certainly have the perception that the number 10 is important to us in our counting systems and everything.

CHRIS BUDD
Yes.

SHANE HUNTINGTON
But we can use other bases for counting that we don?t do, of course, because we have 10 finger and 10 toes.

CHRIS BUDD
Yes.

SHANE HUNTINGTON
Under those sorts of constructs, things do start to look different, don?t they?

CHRIS BUDD
Well, the number 10, yes. It's very much a human number, as it were, because of our fingers and toes. There are other numbers such as the number Pi which is a universal number. If I went anywhere in the universe and measured the ratio of the diameter of a circle to its circumference, I'd still get Pi. That is a universal number that, if I wanted to communicate with aliens that I knew knew mathematics, I'd communicate the number Pi.

SHANE HUNTINGTON
Professor Chris Budd, Professor of Applied Mathematics at the University of Bath and Professor of Mathematics at the Royal Institution of Great Britain. Thank you for being our guest on Up Close today and talking with us about differential equations.

CHRIS BUDD
Thank you.

SHANE HUNTINGTON
Relevant links, a full transcript and more info on this episode can be found on our website at upclose.unimelb.edu.au. Up Close is a production of the University of Melbourne, Australia. This episode was recorded on 11 July 2013. Producers for this episode were Kelvin Param, Eric Van Bemmel and Dr Dyani Lewis. Audio engineering by Gavin Nebauer. Up Close is created by Eric Van Bemmel and Kelvin Param. I'm Dr Shane Huntington. Until next time, goodbye.

VOICEOVER

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